The Study of the Relation between Emission Altitude and Frequency of Pulsars

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Abstract It is important to study the radial structure of pulsar radio emission region; however, the altitude of emission region cannot be derived directly from observation. Usually the altitude can be expressed as a function of frequency, $r_\nu \propto \nu^{-\xi}$. A method of K parameter analysis was applied to calculate the power law index ξ directly from the observational data. The values of ξ are obtained for 18 pulsars at two frequencies of 610 MHz and 1408 MHz. The mean value of the index ξ is 0.27, which indicates that the emission altitude increases with decreasing frequency and that the radial structure is compact.

Key words pulsar-mode change-spectrum

1 Introduction

The radial structure of the emission region of pulsars has been concerned since pulsar was discovered, but even the altitude of emission can not be determined directly from observations.

The radius to frequency mapping model (RFM) was proposed by Cordes (1978) to explain the systematic increase of the separation between the profile components and the profile widths with decreasing frequency. A few methods of calculating radial location of radio emission at different frequencies have been put forward by several authors (Cordes 1978, BCW91, Phi92, KG97, Wu99).

The interval of emission region for frequency 430 MHz and 1400 MHz was found to be less than $\sim 10^2 - 10^3$ km, or a few percent of the light cylinder radius. Phi92 analyzed the timing data of four pulsars. After subtracting the effect of retardation, aberration and magnetic field sweep back, they found that the radial separation of emission region at 47 MHz and 4800 MHz is less than 200 km. BCW91 found that the relativistic electron emission and the rotation of pulsars could result in the delay of the center of linear polarization position angle to the intensity profile center by $\Delta = 4r_{\rm delay}/c$. They obtained the emission altitude $r_{\rm delay}$ at 430 MHz and 1408 MHz for 11 pulsars.

Based on the polar cap model, the altitude of emission can be expressed as a function of period (P) and opening angle (ρ) of the pulsar beam (KG97)

$$\rho = 1.24 s r_6^{1/2} P^{-1/2} \tag{1}$$

where $r_6 = r/R$ is the emission altitude in unit of pulsar radius, s is the parameter that describes a locus of field lines contributing to the emission. $0 < s \leq 1$, $s = 0$ is the axis of the magnetic field, $s = 1$ the last opened field line. In KG's paper, s is assumed to be 1. In order to get the opening angle of emission beams, KG97 used the geometric relation of polar cap

$$\rho = \cos^{-1}[\cos\beta - 2\sin\alpha \sin\theta \sin^2(\triangle\phi/4)]\,, \tag{2}$$

in which $\theta = \alpha + \beta$, α is magnetic inclination angle, β is the angle between the line of sight and magnetic axis, $\Delta\phi$ is the apparent width of mean profile in degree. The emission altitudes can be calculated by equations (1) and (2), the $\Delta\phi$ on different frequency, and θ, α published on journals. KG97 obtained the emission altitude for 430MHz and 1420MHz. They argued that the altitude of radio emission region depends on pulsar period approximately as $P^{0.3}$(KG97). But KG's method is restricted by their adoption of parameters β and α from other literatures. Obviously, the published data of θ and α given by different authors are based on the different $\rho-P$ relations (LM88, Kuzmin 1984, Xu & Wu 1991, R93, G94). If one choose the relationship $\rho \propto P^{-0.5}$, the emission altitude r must be a constant. The value of parameter s in equation (1) also bothers us that only a part of Goldreich-Julian polar cap is involved in emission processes. It means that s is less than 1.

The form of the radius-to-frequency mapping depends on the physics of the emission mechanism. Usually a power-law relationship between emission altitude and frequency is assumed as following

$$r_\nu \propto \nu^{-\xi}\,, \tag{3}$$

where the ξ is a constant. The index ξ is an important parameter to understand the radial structure of emission region in pulsar magnetosphere.

2 The Method of K Parameters Analysis

In order to study the evolution of inclination angle and emission angle, Xu et al (1991) and Wu et al. (1992) defined a parameter K under the polar cap model

$$K = \frac{\sin \triangle\psi}{\sin \triangle\phi} = \frac{\sin\alpha}{\sin\rho}\,, \tag{4}$$

where $\Delta\Psi$ is the half polarization position angle swing, $\Delta\phi$ is the half apparent beam width. K is an observable parameter since both $\Delta\Psi$ and $\Delta\phi$ are observable parameters. α is the inclination angle and ρ is the half angle of emission cone.

The altitude ratio of emission at two frequencies can be obtained by using equation (1) and equation (4):

$$m = \frac{r(f_1)}{r(f_2)} = \frac{\rho^2(f_1)}{\rho^2(f_2)} \approx \frac{K^2(f_2)}{K^2(f_1)}\,. \tag{5}$$

From this formula it is straightforward to get altitude ratio of radio emission of two frequency. The power-law index can be derived :

$$m = \frac{r(610)}{r(1408)} = (\frac{610}{1408})^{-\xi}\,, \tag{6}$$

$$\xi = 2.75\log(\frac{K(1408)}{K(610)})^2 = 2.75\log m\,. \tag{7}$$

The ξ value can be obtained from the K parameters. So the ξ is also determined by observational data. The great advantage of this method is that it does not require to calculate the opening angle ρ of beam cone from the published values of θ and α, and does not need to consider the parameter s in equation (1).

To get the position angle swing $\Delta\Psi$ is much more difficult even for high precision radio observations. For some pulsars the swing of the position angle is an 'S' curve, which can be well understood within the rotating vector model (Radhakrishnan & Cooke, 1969). In the case of 'S' shape, the parameter $\Delta\Psi$ can be measured easily. But in real situations the polarization position angle curves are very complex, such as the position angle jumps. Some curves are asymmetric or lack of a large part of the 'S' curve. An investigation of the relation between the position angle jump and the depolarization due to relative longitude shift of pulsar beams is presented by Xu et al (1997).

For those pulsars with complex polarization position angle curves, the $\Delta\Psi$ cannot be obtained. In this paper we need to know K parameters on two or more frequencies, which means that all of the pulsars in our sample must have good polarization data on at least two frequencies. This strict requirement prevents us from collecting a large database.

The polarization study of mean pulse profile of pulsars is fundamental. Since 1990s, several groups have carried out systematic polarization observations (Wu et al. 1990, 1993, Qiao et al. 1995, Manchester et al, 1998, G94). The largest sample of mean pulse profile and the polarization are presented in the thesis of G94. 18 pulsars with good data of the $\Delta\phi$ and $\Delta\Psi$ at 610 MHz and 1408MHz are selected from G94. The profile boundary cannot be determined precisely, we use the apparent beam width at 10% intensity levels. Rankin (1983) found the phenomena of profile absorption for some pulsars. All these pulsars whose profile might be affected by profile absorption are excluded in our samples.

3 The Power Index ξ of Altitude-Frequency Relation

Table 1 shows the observation parameters at 610 MHz and 1408 MHz and the calculated results of the altitude ratio m and the parameter ξ for 18 pulsars. Obviously the K analyses at more than two frequencies are valuable to understand the property of structure of emission region. However, there are only a few pulsars with good polarization data at three or more than three frequencies.

Table 1 The observed data and derived parameters

PSR B	$\triangle\phi_{610}$	$\triangle\phi_{1408}$	$\triangle\psi_{610}$	$\triangle\psi_{1408}$	K_{610}	K_{1408}	m	ξ
0301+19	18.2	16.3	117.5	122.5	5.41	6.18	1.31	0.32
0525+21	20.4	18.8	153.2	153.2	5.49	5.96	1.18	0.19
0628-28	38.0	35.6	120.3	120.3	2.66	2.84	1.13	0.15
1039-19	19.6	17.7	131.5	142.7	5.36	6.16	1.32	0.33
1133+16	12.1	11.4	88.6	88.6	6.63	7.03	1.13	0.14
1702-19	18.0	17.3	101.3	101.3	4.94	5.14	1.08	0.09
1737+13	23.8	22.0	154.4	177.2	4.73	5.24	1.23	0.24
1811+40	15.7	15.1	139.2	145.6	6.86	7.27	1.12	0.14
1839-04	81.9	70.7	182.3	183.5	1.53	1.73	1.28	0.30
1916+14	9.4	8.0	139.2	139.2	11.4	13.4	1.38	0.38
2011+38	51.1	43.7	31.6	32.9	0.63	0.76	1.45	0.45
2021+51	19.7	16.9	62.0	62.0	3.01	3.50	1.36	0.36
2045-16	17.4	16.1	143.0	158.2	6.27	7.01	1.25	0.27
2154+40	27.5	25.6	126.6	125.3	3.76	4.01	1.14	0.15
2306+55	26.6	24.1	153.2	154.4	4.23	4.67	1.22	0.24
2319+60	26.1	22.9	101.3	101.3	3.42	3.90	1.29	0.31
2323+63	37.0	34.2	183.5	183.5	3.15	3.40	1.16	0.18
2324+60	31.3	26.4	145.6	177.2	3.54	4.38	1.53	0.51

All of the 18 pulsars have $m > 1$ and $\xi > 0$, which indicate that the emission region of 610 MHz is higher than the emission region of 1408 MHz. This conclusion is in agreement with RFM model. The mean value of m is 1.25±0.12, from 1.08 to 1.53; the power law index (ξ) of altitude-frequency ($r_\nu \propto \nu^{-\xi}$) is 0.27±0.12, from 0.09 to 0.51. These estimates show that the pulsar emission region is very compact.

Theoretically the value of the spectral index ξ is a constant from $\xi = 0$ (Barnad & Arons 1986) to $\xi = 2/3$ (Ruderman & Sutherland 1975). The results of 18 pulsars in this paper are clearly desirable to discriminate the competing theory, and our results consistent with the assumption of dipolar field in the radio emission zone.

The average value ξ we derived in this paper is comparable to those obtained by using other methods (Table 2). All those values of ξ given by this paper and other papers listed in Table 2 are less than 0.66, which is an upper limit of ξ given by Phi92.

Table 2 The comparison of the ξ given by this paper and other papers

0.21±0.1	0.43-1.42	11	BCW91
≤0.66	0.05-4.80	4	Phi92
0.12±0.08	0.43-1.42	6	GK93
0.11±0.02	1.41-10.6	5	K94
0.21±0.07	0.43-1.42	10	KG97
0.26±0.09	0.43-1.42	16	KG98
0.21±0.1	0.43-1.42	8	Wu99
0.27±0.12	0.61-1.41	18	This paper

For some individual pulsars, the relations between emission altitude and frequencies were obtained. For PSR1451-68 at 0.17-1.62 GHz: $r(\mathrm{km}) = 225 f_{\mathrm{GHz}}^{-0.37}$ (Wu et al. 1998); for PSR 1857-26 at 0.17—2.65 GHz: $r(\mathrm{km}) = 249 f_{\mathrm{GHz}}^{-0.30}$ (R93); for PSR 2111+40 at 0.408—4.85 GHz.

4 Summary and Discussion

The method of the K parameter analyses was used to calculate the power law index ξ of altitude-frequency relation ($r_\nu \propto \nu^{-\xi}$) directly from observational data at different frequencies. The values of ξ are obtained for 18 pulsars at two frequencies of 610 MHz and 1408 MHz and for 3 pulsars at more than three frequencies. The average of the index ξ is 0.27, which is consistent with those obtained using different estimation methods (Table 2). The emission altitude increases with decreasing frequency, and the radial structure is compact.

The altitudes of emission region at 610 MHz and 1408 MHz are calculated using the method of R93 and GK93. The values of α and β were calculated using the new data of apparent beam width ($\Delta\phi_{10}$) according the method of LM88. The averaged value of emission altitude is 290.1±100.6 km above the surface of neutron star at 610 MHz, and 236.4±86.4 km at frequency of 1408 MHz. The total size of radio emission regions in the range between 0.61 and 1.408 GHz is only 54.2±29.2 km.

There is close relation between the emission altitude and pulsar period, which is in agreement with the results of GK93 and KG97. But the method of calculating α, β may affect these results of the relation between the altitude and period, due to the $\rho - P$ relations introduced into their simultaneous equations, such as $\rho \propto P^{-0.33}$ at 400 MHz of LM88. If such relation of $\rho \propto P^{-0.33}$ is applicable to different frequencies, then we can get the relation $r \propto P^{0.33}$ from equation (1), which is similar with the results of $r \propto P^{-0.38}$ at 610 MHz and $r \propto P^{-0.36}$ at 1408 MHz given by this paper.

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