A Study of Optimization of the Focus in the FAST

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Abstract FAST will be a Five-hundred-meters Aperture Spherical Telescope built in Guizhou Province, China. The active panels can be simulated to a parabola in time during tracking a source. It is important to choose an optimal focus to ensure a minimum deviation from a spherical panels to the parabola, a minimum relative displacement of panels for the simulating and feasibility of utilization of multi-parabola fitting scheme. An optimal focus could also reduce problems of collision or unnecessarily larger gaps between panels in simulation, realization of multi-beam scheme and decrease errors in programming and adjusting the panels. A study of optimization of focus in the FAST is introduced in this paper.

Key words pulsar-mode change-spectrum

1 The Deviation Between Two Co-apex Surfaces

In a Cartesian coordinate originated at the apex of both the spherical surface and the parabola simulated, the cross sections of these surfaces can be expressed in ordinate direction by following 2-D equations:

$$S(x) = R - \sqrt{(R^2 - x^2)}; \qquad P(x, f) = x^2/4fR\,. \tag{1}$$

Their centripetal discrepancy and the standard deviation can be expressed as:

$$D(x, f) = [P(x, f) - S(x)] \cos\left(\arcsin\left(\frac{x}{R}\right)\right) \quad \text{and} \quad \sigma_c(f) = \left[\sum_{x=0}^{150} D(x, f)^2 \frac{1}{150}\right]^{0.5} \tag{2}$$

respectively, the latter can be indicated as Fig. 1. The best focus should be about 0.476 as shown in the figure (Qiu 1998).

Of course formula (2) is only an approximation for simplifying the calculation and the analysis mentioned in [1]. The standard deviation deduced from a set of exact formulae of the discrepancy could be exactly presented in the same figure as $\sigma_e(f)$.

A slender difference between the approximation and the exact expression might be negligible, but could be shown clearly when $f > 0.475$ by comparing corresponding curves. In fact, the standard deviation of $D(x, f)$ can be further calculated as $\sigma_a(f)$and indicated as a circled-dashed curve in Fig. 1. The minimum of the latter appears at $f = 0.472$, that should be the best choice in view of minimum deviation between spherical surface and parabola simulated. There the average cosine discrepancy is also shown by a dashed straight line.

2 Minimum relative displacement of panels

If a neutral spherical surface is preferred for the programming and adjustment of the surface, it is desired to choose a focus for ensuring minimum relative displacement from the

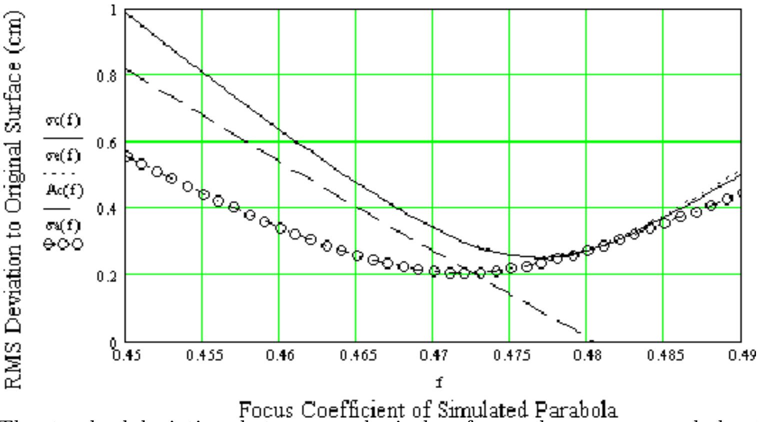

Fig. 1 The standard deviations between a spherical surface and a co-apex parabola within an aperture of 300m calculated with both cosine approximation and exact formula respectively

neutral surface. A 3-D plot of $D(x, f)$ can be drawn (omitted here) and used to determine a focus coefficient, $f = 0.4725$, to ensure relative displacement less than ± 0.46 m. The relative displacement would be less than ± 0.5m if the focus coefficient was 0.47.

In contrast with the former, the focus coefficient could be determined by a boundary condition of the illuminated aperture, D(150,f)=0, as expressed in the Eq. (2), if one-direction adjustment of the panels was preferred. Then we would have f =0.4665, the maximum displacement would be some +0.7 m.

3 A weighted average of the deviation

It is important to select a focus to ensure minimum panels fitting from spherical contour to desired parabolic contour, because the spherical surface consists of more than one thousand panels in size of about 12 m (Qiu 1998). We could probably use a weighted average of the deviation, $\overline{\sigma(f)}$, to indicate the quality of the panels fitting vs. focus coefficient f. For instance, taking account of the reflector area and a square cosine illumination function, a weighted factor $W(m) = \frac{[2\cos\phi(m)-1]^2(\sin 2\phi(m))^{0.5}}{5-4\cos\phi(m)}$ can be introduced. The weighted average of the deviation can be calculated as follows:

$$\overline{\sigma(f)} = \sum_m \sigma(m, f)W(m)/\sum_m W(m)\,. \tag{3}$$

In the formula, the viewing angle of area element from the curvature center of the main mirror relative to the major optical axis is expressed as: $\phi(m) = 0.2^o \times m$.

Fig. 2 shows the weighted average as a function of the focus coefficient. It is obvious that $\overline{\sigma(f)}$ would increase slowly with the focus ratio, within $0.45 \leq f \leq 0.48$. That means we should avoid to choose the focus coefficient considerably larger than 0.474 if we did not want the average deviation larger than 0.185 cm.

4 Differences of radial arcs and areas between the surfaces

The lengths of radial arcs of the spherical surface and simulated parabola from the apex to the edge of the 300 m aperture and the areas of corresponding surfaces can be calculated as $L_s(f), L_p(f), A_s(f)$ and $A_p(f)$ respectively from basic infinitesimal calculus. Then their

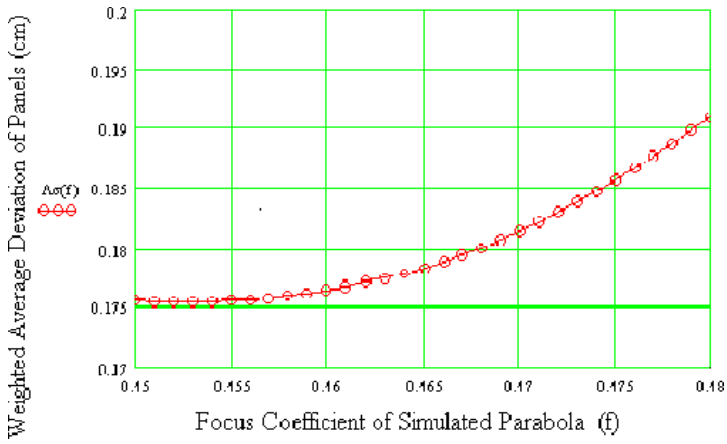

Fig. 2 The weighted average of the standard deviation of spherical panels to parabola vs. focus coefficient between 0.45and 0.48

discrepancies of the radial arcs $d(f) = L_s(f) - L_p(f)$ and the areas $D(f) = A_s(f) - A_p(f)$ within 300 m aperture can be drawn as a figure (omitted here). The discrepancies increase monotonously with the focus coefficient. It is obvious that $d(f) = 0$ or $D(f) = 0$ if the focus coefficient $f = 0.46$ or 0.456. In reality, the focus coefficient f should be mildly larger than 0.46, if panels would not collide or overlap with each other in tracking a radio source. A trade off must be made for leaving reasonable gaps between panels in choices of the focus coefficient, the size and shape of panels. For instance the average width of radial gaps and gaps all over the main surface can be statistically calculated on the assumption of some 10 m rectangle panels or 15 m hexagon panels being used. The results of the calculation are listed in Table 1. In fact, all gaps on the main spherical surface should be left around 2.8 cm if f = 0.47 was adopted finally.

Table 1 Average width of radial gaps and all gaps on the main surface in different conditions

Focus f	Discr.Arc d(cm)	Discr.Area D(m^2)	10 m Rect. W_{rg}(cm)	10 m Rect. W_{alg}(cm)	15 m Hexag W_{rg}(cm)	15 m Hexag W_{alg}(cm)
0.467	20	230	1.43	1.87	1.9	1.72
0.469	27.3	281.7	1.95	2.29	2.6	2.1
0.470	28	291.5	1.99	2.37	2.7	2.16

5 Minimum Errors of the Cosine Approximation

In the analysis of the FAST appeared in paper [1], an approximate factor $\cos(\arcsin(x/R))$ was used to transfer all displacement in ordinate into the centripetal direction. The discrepancy of the approximation (2) and the exact expressions causes errors in programming panels that can be indicated in Fig. 3. It would be very important to minimize the error if the telescope was hopefully operated at short centimeter wavelengths. The best choice of the focus seems to be f = 0.47 in this meaning, that would ensure minimum error (less than ±3 mm within an aperture of 300 m) in panels programming if the simple cosine factor mentioned was still used. A minimum RMS error appears around this focus ratio that would ensure the RMS error less than 0.12 cm.

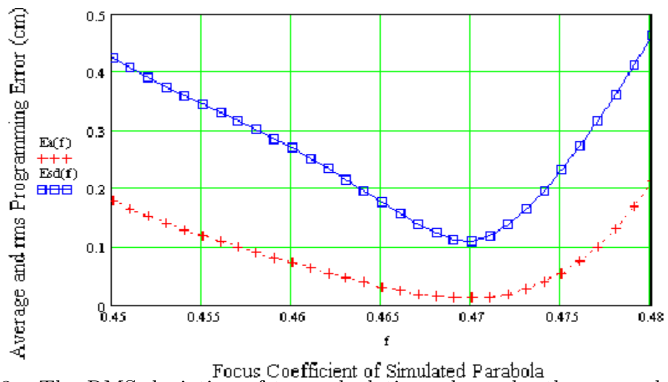

Fig. 3 The RMS deviation of two calculations shown by the squared-solid curve and the average square discrepancy indicated by the crossed-dashed curve

6 Possibility of Realizing Multi-parabola Fitting

It is required to make use of a scheme of dual-parabola fitting in the FAST project in order to extend the observable zenith angle up to a half of the opening angle of the main spherical surface (Wu 2000) or even more. One of essential technical difficulties associated with this scheme is that a quite large displacement between the original and additional feeds (or feed systems) is needed to fit an outside parabolic ring. A relation of the required displacement and the focus coefficient can be deduced and shown in Fig. 4.

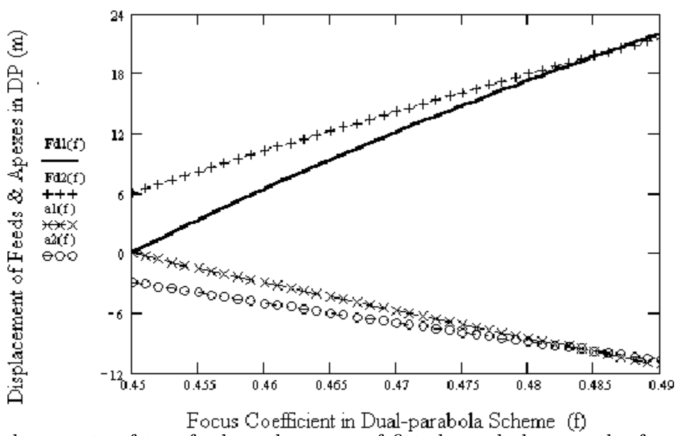

Fig. 4 Displacements of two feeds and apexes of fitted parabolas vs. the focus coefficient between 0.45 and 0.49 in the dual-parabola fitting scheme

The displacements of two feed systems and the apexes with respect of 0-150 m (inner parabola) and 150-180 m (outer parabolic ring) are indicated as $Fd1(f)$ (solid curve) and $a1(f)$ (x-curve)respectively. They are $Fd2(f)$ (crossed curve) and $a2(f)$(circled curve) for the case of 0-150 m and 150-200 m in the figure.

It is obvious that the displacements increase with focus coefficient from f = 0.45. So too large focus coefficient should not be chosen in this respect. On the other hand, no essential adjustment is required when the scheme of offset illumination (Wu 1999) is used to enlarge the sky coverage of the FAST, except the control of the feed inclination should be programmed using a bit different formula.

7 Application of a FPA or MBF to the FAST

The focus plane array (FPA) or the multi-beam feed (MBF) is increasingly widely used on radio telescopes to fasten observations of extended radio sources or to exclude any harmful influence of atmospheric irregularity and defects of radio telescopes.

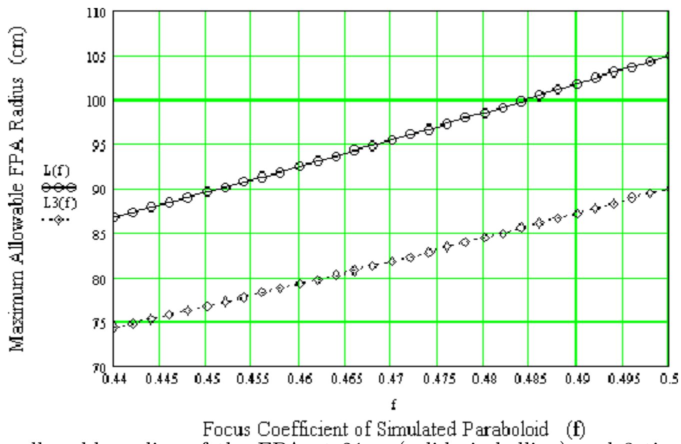

Fig. 5 The allowable radius of the FPA at 21 cm(solid circledline) and 3 times allowable radius at 6 cm (dashed diamond line) vs. the focus coefficient of the FAST

The size of an imageable field on the focus plane is restricted by allowable phase error at the edge of the field (Wu 2001). That is really a key point in designing,installing the FPA and in choice of the focus ratio of telescopes.

The permissible radius of a FPA could be calculated as:

$$L = \frac{8\lambda}{f}(f^2 + 0.25)(2f^2 + 0.125)\,, \tag{4}$$

L increases quickly with the focus coefficient f in this case. So as large as possible f should be used in the FAST. Fig. 5 shows the radius of a FPA at 21 cm $L(f)$ and three time that at 6 cm $L3(f)$ varies with f. In reality, L changes only a little within a reasonable range of the focus coefficient. For instance, $L_{21} = 94$ cm and $L_6 = 26.8$ cm if $f = 0.465$, $L_{21} = 95$ cm and $L_6 = 27.3$ cm if $f = 0.47$.

8 Conclusions

The focus coefficient can be chosen within a range to achieve a reasonably good performance of the FAST from all aspects mentioned above. In reality there is no solely analytic solution, e.g. a weighted average, so a trade off has to be made to get an almost optimization.

Considering all items mentioned, the focus coefficient of 0.47 is finally suggested. In this case, the standard deviation of the spherical surface and parabola simulated is 0.2 cm, relative displacement from the neutral surface is less than 0.5 m, weighted average of the RMS panels fitting is 0.182 cm. An average gaps between panels is about 2.7 cm, maximum and RMS programming errors are 0.55 and 0.11 cm respectively. The displacement of feeds (or feed systems) would be 12 m or 14 m if the dual-parabola fitting scheme of 150-180 m or 150-200 m were adopted. The radius of a FPA would be 95 cm and 27.3 cm respectively at $\lambda = 21$ and 6 cm. This optimization of the focus coefficient is necessary to ensure the FAST operating at shorter centimeter wavelengths. If $f = 0.467$ (or 0.4665) was used to simplify the action of the actuators only in one direction, then corresponding values would be 0.23 cm, +0.7 m, 0.178 cm, 2 cm, 0.35 cm, 0.14 cm, 10 m and 13 m, 94 and 26.8 cm in the same order.

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