

# Neutron Star Interiors and Pulsar Manifestations

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# Outline

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- 1、 Introduction
- 2、 Surface temperature observations of NSs and the cooling theory
- 3、 NS cooling with the heating due to r-mode damping
- 4、 The rapid cooling of Cassiopeia A NS and its explanations
- 5、 Summary

# 1、 Introduction

## Pulsar observations

- 1) Mass
- 2) Rotation frequency
- 3) Surface temperature

## Physics in NS interiors

Equation of state

Viscosity of cold dense matter

Thermal physics

Heat capacity

Neutrino emission rates

Heating mechanisms:

(rotochemical heating,

Joule heating,

heating due to r-mode damping,  
etc.)

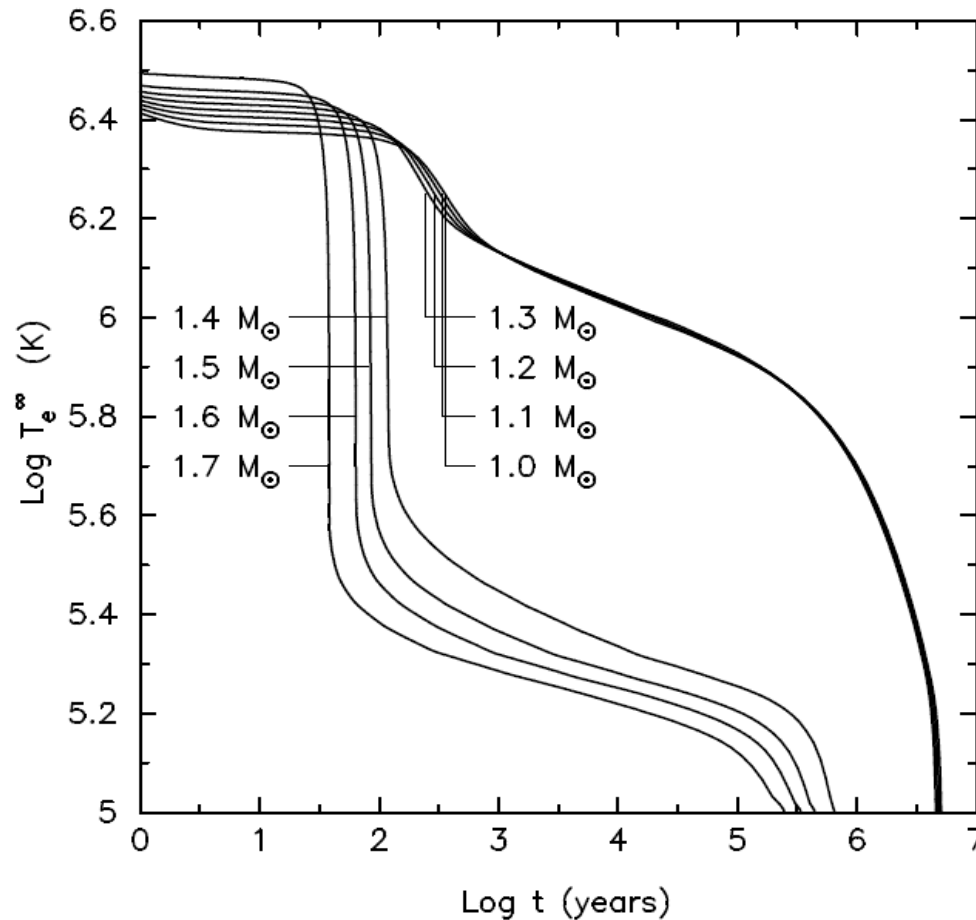
## 2、 Surface temperature observations of NSs and the cooling theory

TABLE 3. Observational limits on surface temperatures of isolated neutron stars

Number	Source	$t$ [kyr]	$T_s^\infty$ [MK]	Confid.	Model	Ref.
1	PSR B0531+21 (Crab)	1	$< 2.0$	99.8%	BB	[37]
2	PSR J0205+6449 (in 3C 58)	0.82–5.4	$< 1.02$	99.8%	BB	[38]
3	PSR J1119–6127	$\sim 1.6$	$\approx 1.2$	–	mHA	[32]
4	RX J0822–4300 (in Pup A)	2–5	1.6–1.9	90%	HA	[39]
5	PSR J1357–6429	$\sim 7.3$	$\approx 0.766$	–	mHA	[40]
6	RX J0007.0+7303 (in CTA 1)	10–30	$< 0.66$	–	BB	[41]
7	PSR B0833–45 (Vela)	11–25	$0.68 \pm 0.03$	68%	mHA	[42]
8	PSR B1706–44	$\sim 17$	$0.82^{+0.01}_{-0.34}$	68%	mHA	[43]
9	PSR J0538+2817	$30 \pm 4$	$\sim 0.87$	–	mHA	[44]
10	PSR B2334+61	$\sim 41$	$\sim 0.69$	–	mHA	[32]
11	PSR B0656+14	$\sim 110$	$0.91 \pm 0.05$	90%	BB	[45]
12	PSR B0633+1748 (Geminga)	$\sim 340$	$\sim 0.5$	–	BB	[46]
13	RX J1856.4–3754	$\sim 500$	$0.434 \pm 0.003$	68%	mHA*	[47]
14	PSR B1055–52	$\sim 540$	$\sim 0.75$	–	BB	[48]
15	PSR J2043+2740	$\sim 1200$	$\sim 0.44$	–	mHA	[32]
16	RX J0720.4–3125	$\sim 1300$	$\sim 0.51$	–	HA*	[49]

# The thermal evolution equation with the approximation of isothermal interior

$$C_V \frac{dT}{dt} = -L_\nu - L_\gamma + H$$

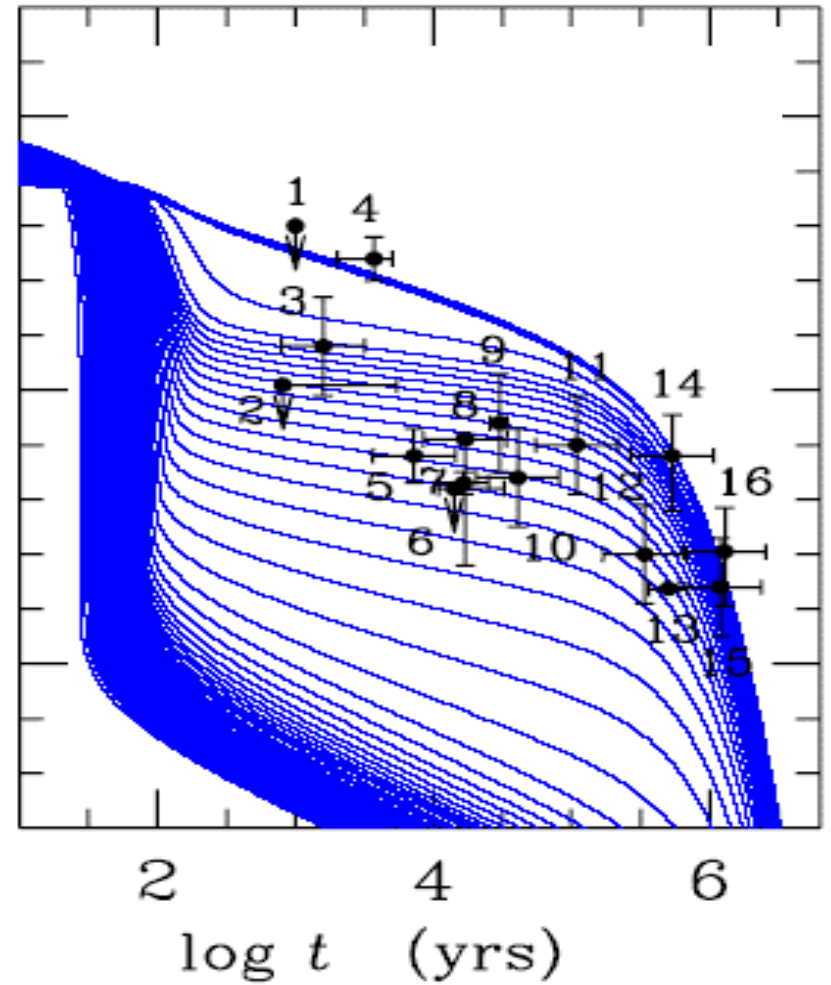
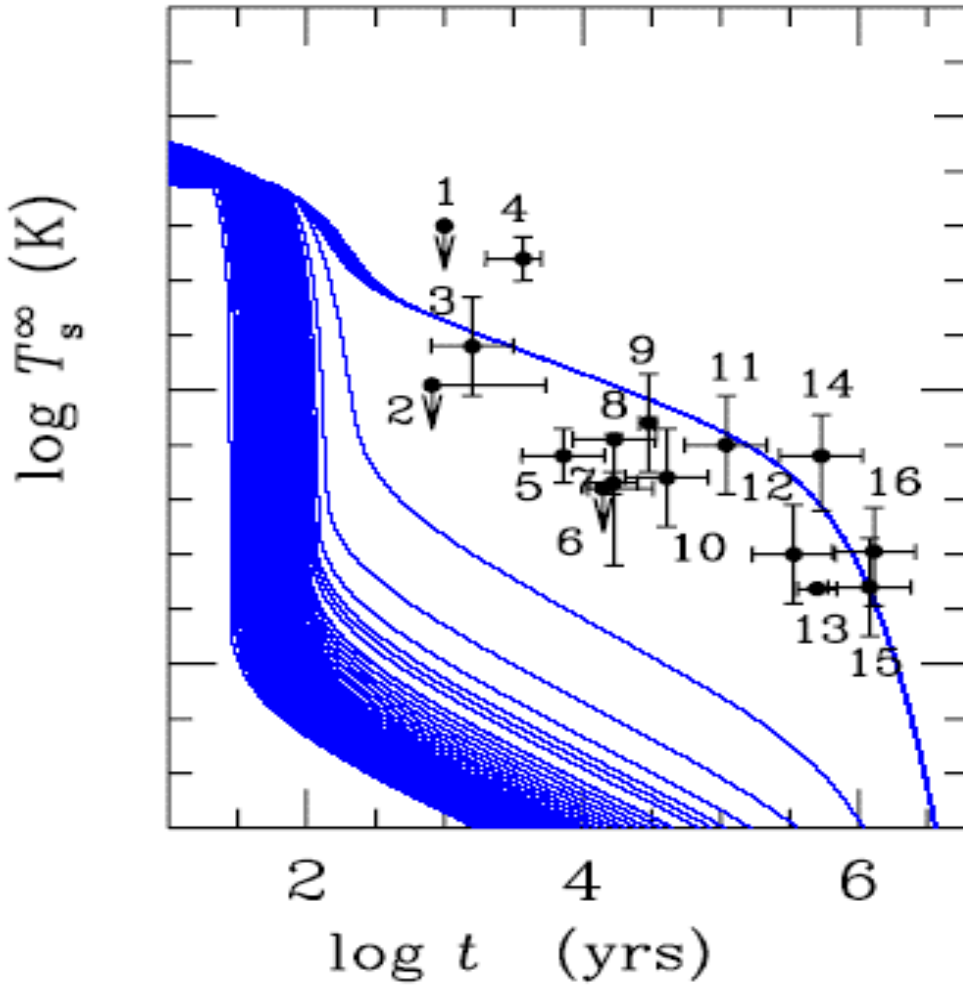


## Slow and fast neutrino emission processes

Process	$Q_s, \text{ erg cm}^{-3} \text{ s}^{-1}$
Modified Urca $nN \rightarrow pNe\bar{\nu}$ $pNe \rightarrow nN\nu$	$10^{20} - 3 \times 10^{21}$
Bremsstrahlung $NN \rightarrow NN\nu\bar{\nu}$	$10^{19} - 10^{20}$

Model	Process	$Q_f, \text{ erg cm}^{-3} \text{ s}^{-1}$
Nucleon matter	$n \rightarrow pe\bar{\nu}$ $pe \rightarrow n\nu$	$10^{26} - 3 \times 10^{27}$
Pion condensate	$\tilde{N} \rightarrow \tilde{N}e\bar{\nu}$ $\tilde{N}e \rightarrow \tilde{N}\nu$	$10^{23} - 10^{26}$
Kaon condensate	$\tilde{B} \rightarrow \tilde{B}e\bar{\nu}$ $\tilde{B}e \rightarrow \tilde{B}\nu$	$10^{23} - 10^{24}$
Quark matter	$d \rightarrow ue\bar{\nu}$ $ue \rightarrow d\nu$	$10^{23} - 10^{24}$

$$Q_{\text{slow}} = Q_s T_9^8, \quad Q_{\text{fast}} = Q_f T_9^6$$



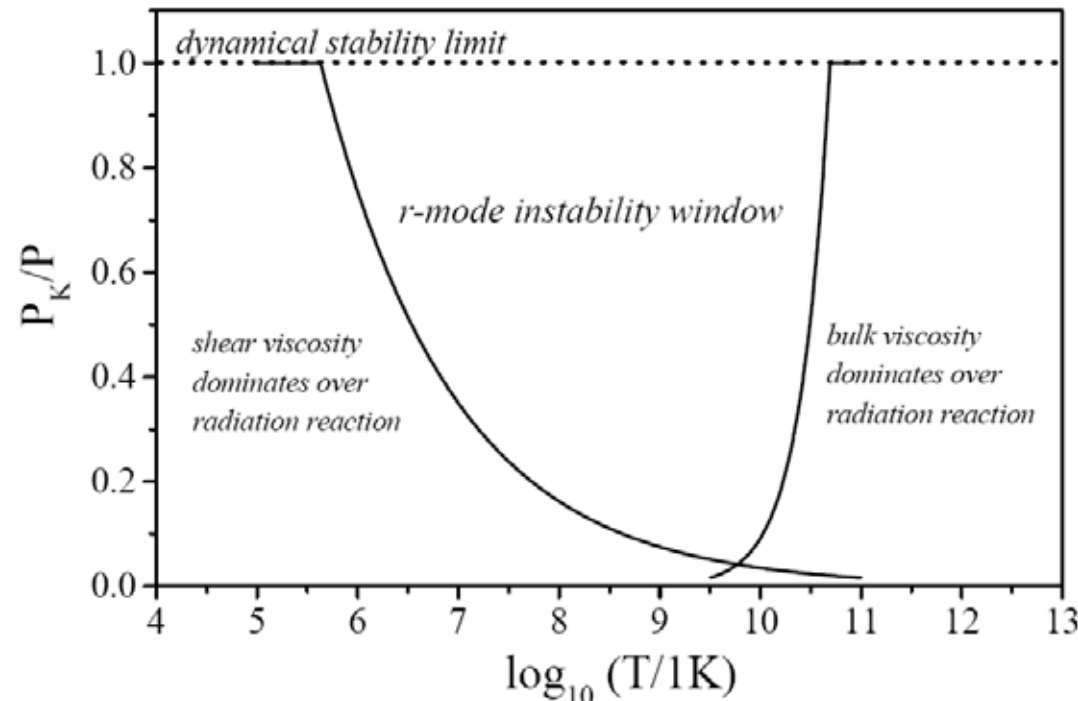
**No superfluidity**

**Proton superfluidity**  $T_{\text{op}}^{\text{max}} = 6.8 \times 10^9 \text{ K}$

*Yakovlev et al. (2008) , AIPC,983,379*

### 3、 NS cooling with the heating due to the r-mode damping

- ◆ R-mode in a perfect fluid star with arbitrary rotation due to the action of the Coriolis force with positive feed back, succumbing to CFS instability.
- ◆ In contrast, the growth of the modes can be suppressed by the viscosity of the stellar matter.



$$\frac{1}{2E} \frac{dE}{dt} = \frac{1}{t_{\text{gw}}} + \sum \frac{1}{t_{\text{diss}}} = 0$$

$$\frac{1}{t_{\text{diss}}} = \frac{1}{t_{\text{sv}}} + \frac{1}{t_{\text{bv}}} + \text{other dissipation terms}$$

Andersson et al (2001)



# R-mode evolution in neutron stars

The r-modes of rotating barotropic Newtonian stars are solutions of the perturbed fluid equations having velocity perturbations

$$\delta \mathbf{v} = \alpha R \Omega \left( \frac{r}{R} \right)^l \mathbf{Y}_{lm}^B e^{i\omega t}$$

where  $\Omega$  is angular velocity of the unperturbed the star,  
 $\alpha$  is the dimensionless amplitude of the perturbation,  
 $\mathbf{Y}_{lm}^B$  is the magnetic-type vector spherical harmonic:

$$\mathbf{Y}_{lm}^B = [l(l+1)]^{-1/2} r \nabla \times (r \nabla Y_{lm})$$

- ◆ First-order r-modes (Owen et al. 1998)

In spherical coordinates, solving the linear fluid equations at the first order of the r-mode amplitude ( $\alpha$ ), we can get the r-mode solutions:

$$\delta^{(1)}v^r = 0, \quad (1a)$$

$$\delta^{(1)}v^\theta = \alpha\Omega C_l l \left(\frac{r}{R}\right)^{l-1} \sin^{l-1}\theta \sin(l\phi + \omega t), \quad (1b)$$

$$\delta^{(1)}v^\phi = \alpha\Omega C_l l \left(\frac{r}{R}\right)^{l-1} \sin^{l-2}\theta \cos\theta \cos(l\phi + \omega t), \quad (1c)$$

## ◆ Second-order r-modes

At the second order of the r-mode amplitude (Sa 2004)

$$\delta^{(2)}v^r = 0, \quad (10a)$$

$$\delta^{(2)}v^\theta = 0, \quad (10b)$$

$$\begin{aligned} \delta^{(2)}v^\phi = & \frac{1}{2} \alpha^2 \Omega C_l^2 l^2 (l^2 - 1) \left(\frac{r}{R}\right)^{2l-2} \sin^{2l-4} \theta \\ & + \alpha^2 \Omega A r^{N-1} \sin^{N-1} \theta, \end{aligned} \quad (10c)$$

A and N are two constants determined by the initial condition.

This second-order solution gives a **differential rotation**, producing large scale drifts of fluid elements along stellar latitudes.

Sa & Tome (2005) suggested  $N = 2l - 1$  and redefined A by introducing a **new free parameter K** as

$$A = \frac{1}{2} K C_l^2 l^2 (l + 1) R^{2-2l}$$

The physical angular momentum of the  $l=2$  r-mode calculated up to the second order is **(Sa & Tome 2005)**

$$J_r = J^{(1)} + J^{(2)} = \frac{(4K + 5)}{2} \alpha^2 \tilde{J} M R^2 \Omega,$$

For  $K = -2$ ,  $J^{(2)} = 0$ ,  $J_r$  return to the first order case.

$$\frac{dJ_r}{dt} = \frac{2J_r}{\tau_g} - \frac{2J_r}{\tau_v}$$

$$\tau_v = (\tau_{sv}^{-1} + \tau_{bv}^{-1})^{-1}$$

$$\frac{dJ}{dt} = -\frac{3\alpha^2 \tilde{J} M R^2 \Omega}{\tau_g} - \frac{I\Omega}{\tau_m}$$

$J = I\Omega + J_r$  is the total angular momentum of the star,

$\tau_m$  is the magnetic braking timescale.

## R-mode evolution

$$\frac{d\alpha}{dt} = \left[ 1 + \frac{4}{3}(K + 2)Q\alpha^2 \right] \frac{\alpha}{\tau_g} - \left[ 1 + \frac{1}{3}(4K + 5)Q\alpha^2 \right] \frac{\alpha}{\tau_v} + \frac{\alpha}{2\tau_m}$$

## Spin evolution of NSs

$$\frac{d\Omega}{dt} = -\frac{8}{3}(K + 2)Q\alpha^2 \frac{\Omega}{\tau_g} + \frac{2}{3}(4K + 5)Q\alpha^2 \frac{\Omega}{\tau_v} - \frac{\Omega}{\tau_m}$$

## Thermal evolution of NSs

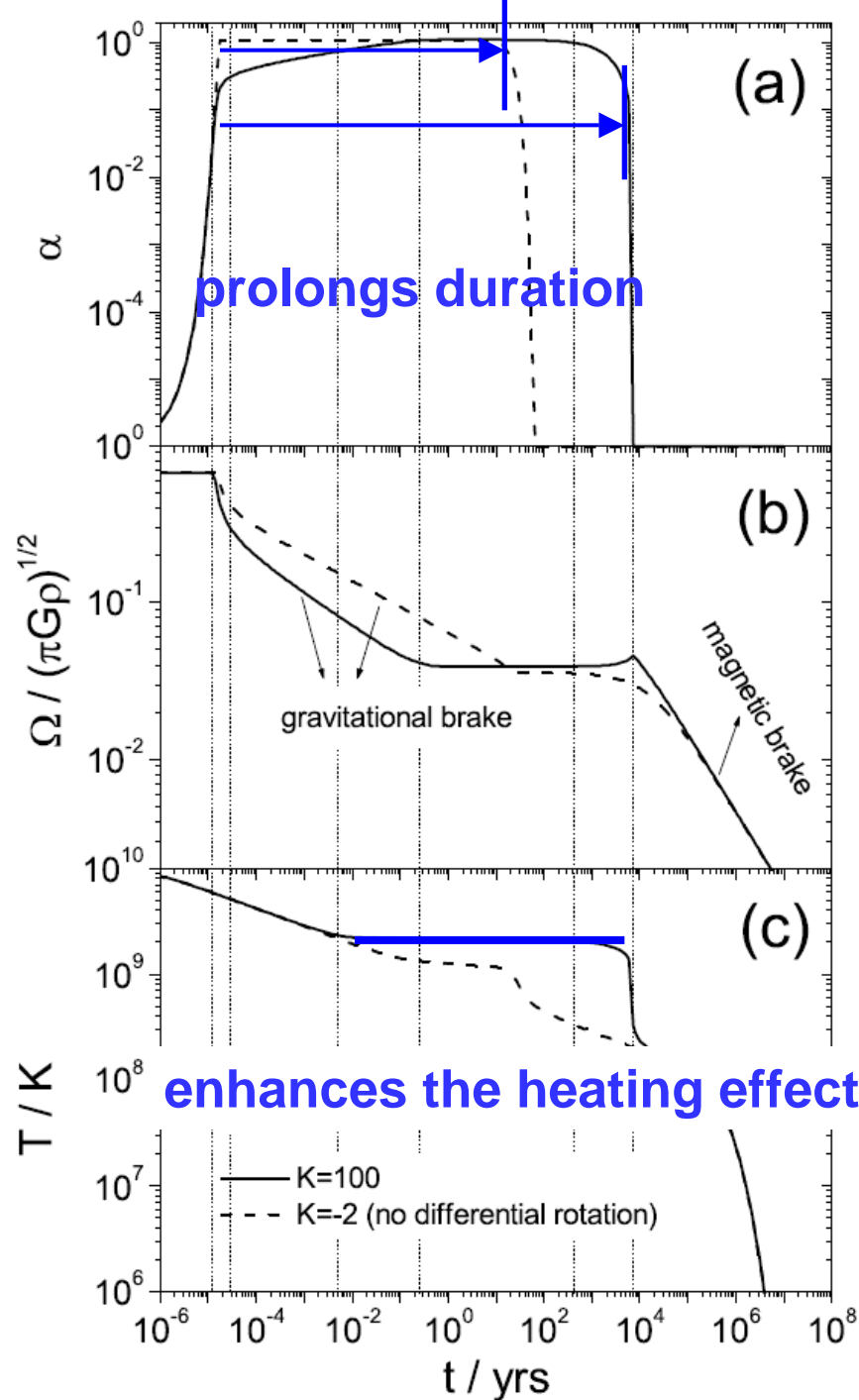
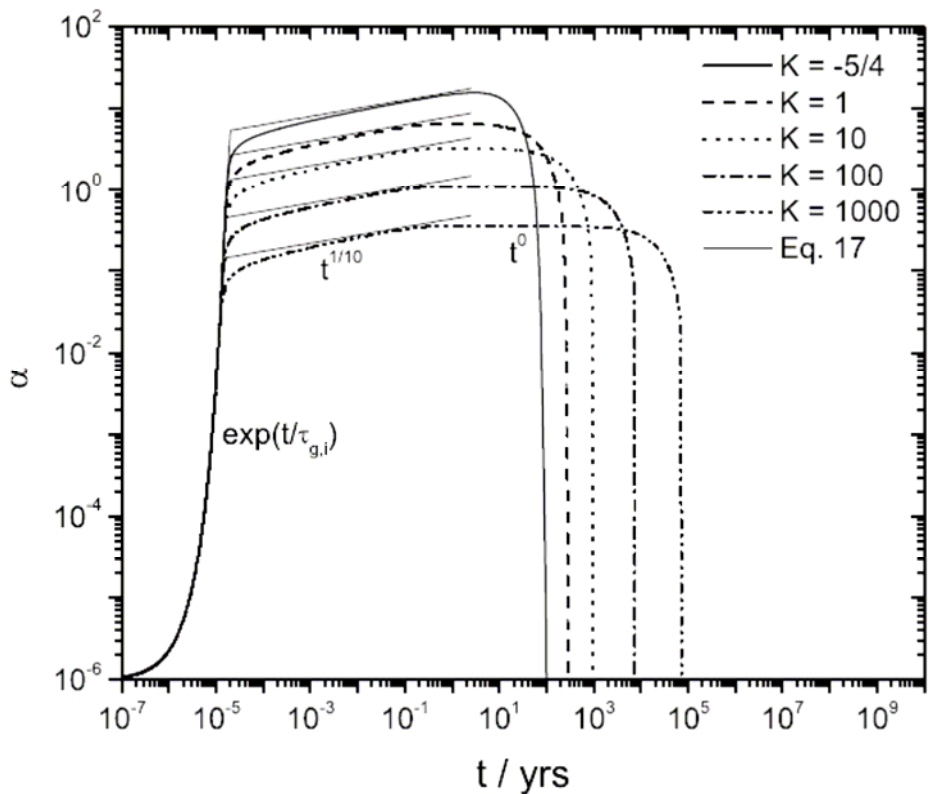
$$C_V \frac{dT}{dt} = -L_\nu - L_\gamma + H_\nu$$

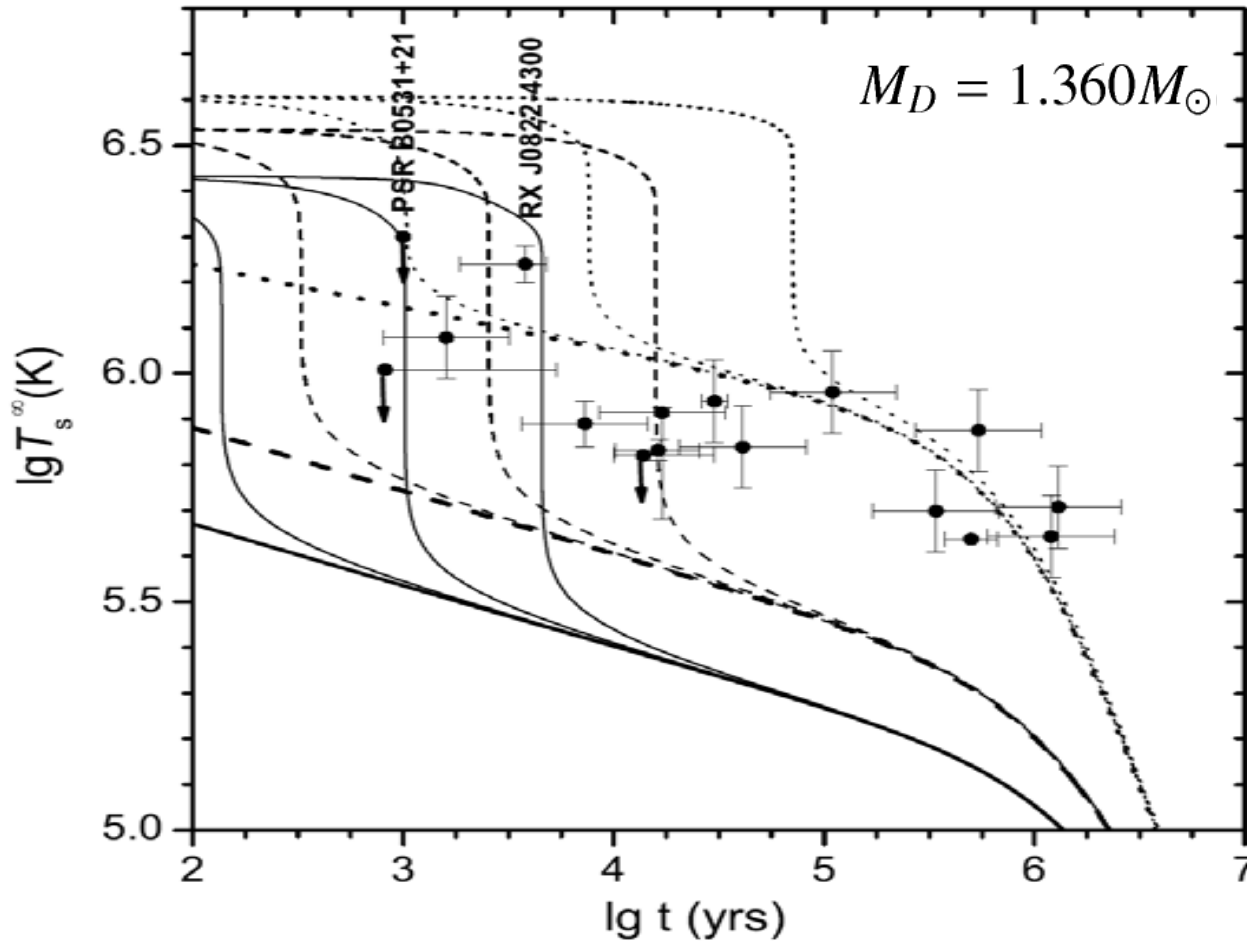
$$H_\nu = 2E_r \left( \frac{1}{\tau_{sv}} + \frac{1}{\tau_{bv}} \right)$$

# Long-term evolution of isolated NSs

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The dot, dashed and solid curves correspond to  $M = 1.3M_\odot$ ,  $M = 1.365M_\odot$  and  $M = 1.4M_\odot$ , respectively. The thick lines are calculated without the  $r$ -mode dissipation effect, and the thin lines refer to  $K = 10, 100, 1000$  from left to right.

## Conclusions:

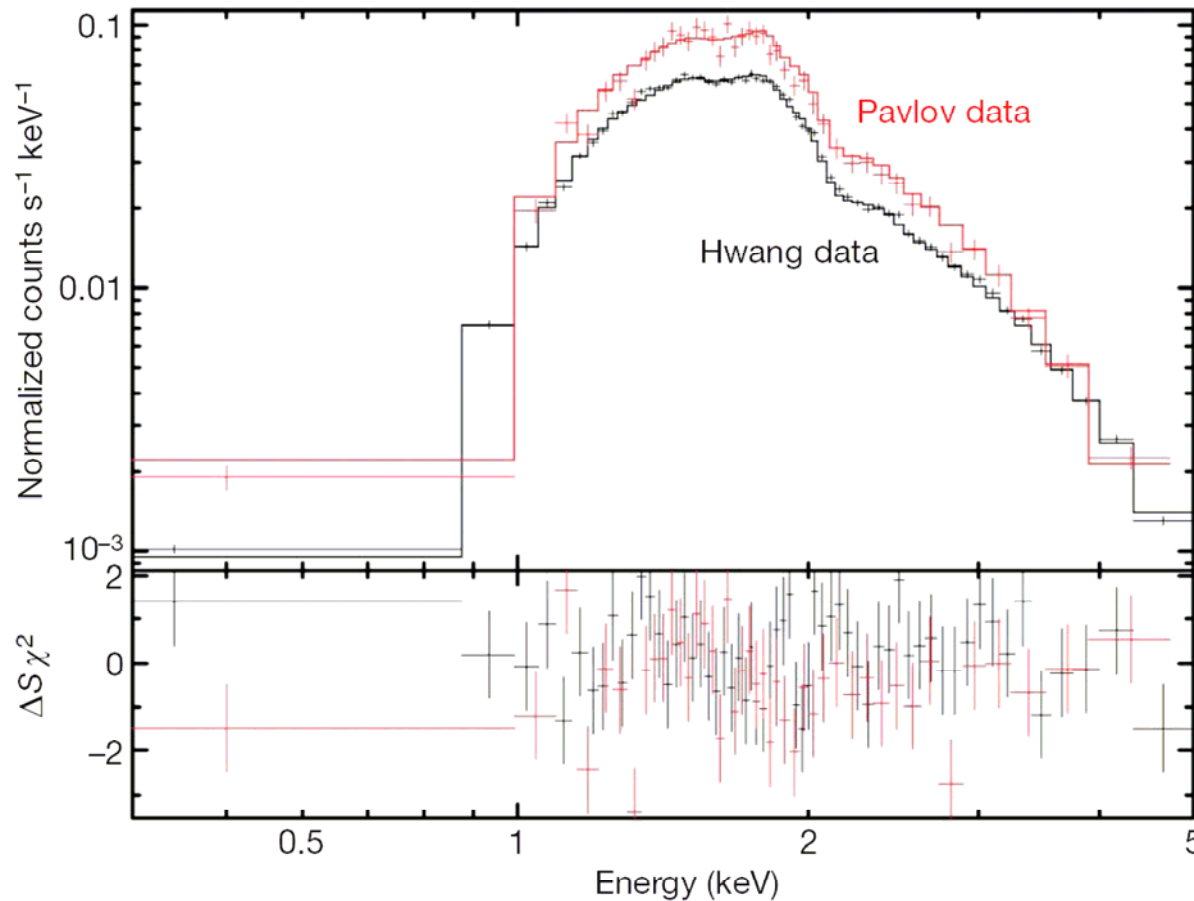
(1) The NS is heated due to shear viscous damping of  $r$ -modes, and it can keep a high temperature for several thousand years, even tens of thousands of years.

This enables us to explain two young and hot pulsar data (PSR B0531+21 and RX J0822-4300) with NS model composed of only  $npe$  matter, without superfluidity or exotic particles.

(2) If consider a wider value range of NS mass and  $K$ , our light curves may probably cover all of the young and middle-aged thermal emission data, and the artificially strong  $p$  superfluidity invoked in Kaminker et al. (2001) is no longer needed.



## 4、 The rapid cooling of Cassiopeia A NS and its explanations



**Figure 1 | Chandra X-ray spectra of Cas A.** Spectra from the Hwang (black) and Pavlov (red) observations and fits with our C spectral model. Error bars

**Table 1.** Carbon atmosphere spectral fits, using the best spectral fit ( $M$ ,  $R$ ,  $N_{\text{H}}$ ) of Heinke & Ho (2010) and Yakovlev et al. (2011), with the addition of 2010 data. Epoch dates are for the midpoints of the observations, or weighted midpoints of merged datasets. Temperature errors are  $1\sigma$  confidence for a single parameter.

Epoch (Year)	Exposure ks	$\log T_s$ K	ObsID(s)
2000.08	50.56	$6.3258^{+0.0019}_{-0.0019}$	114
2002.10	50.3	$6.3237^{+0.0018}_{-0.0018}$	1952
2004.11	50.16	$6.3156^{+0.0019}_{-0.0019}$	5196
2007.93	50.35	$6.3108^{+0.0019}_{-0.0019}$	9117, 9773
2009.84	46.26	$6.3087^{+0.0018}_{-0.0018}$	10935, 12020
2010.83	49.49	$6.3060^{+0.0019}_{-0.0018}$	10936, 13177

**Age of Cas A:**

$t \approx 330 \pm 20$  yr

**Rapid cooling of about 4% in 10 years!**

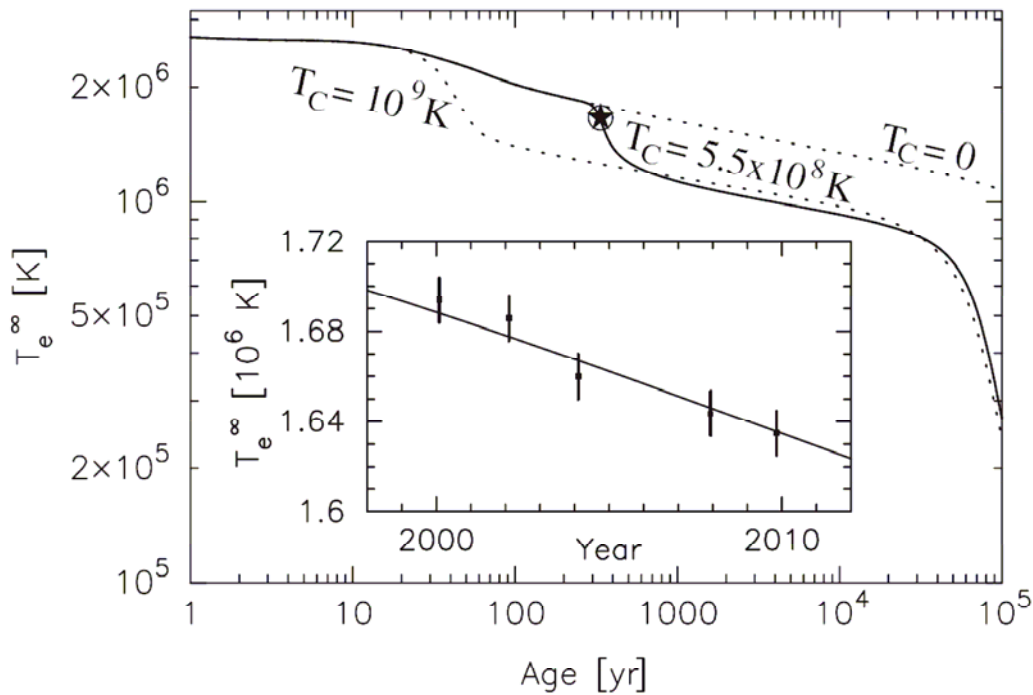
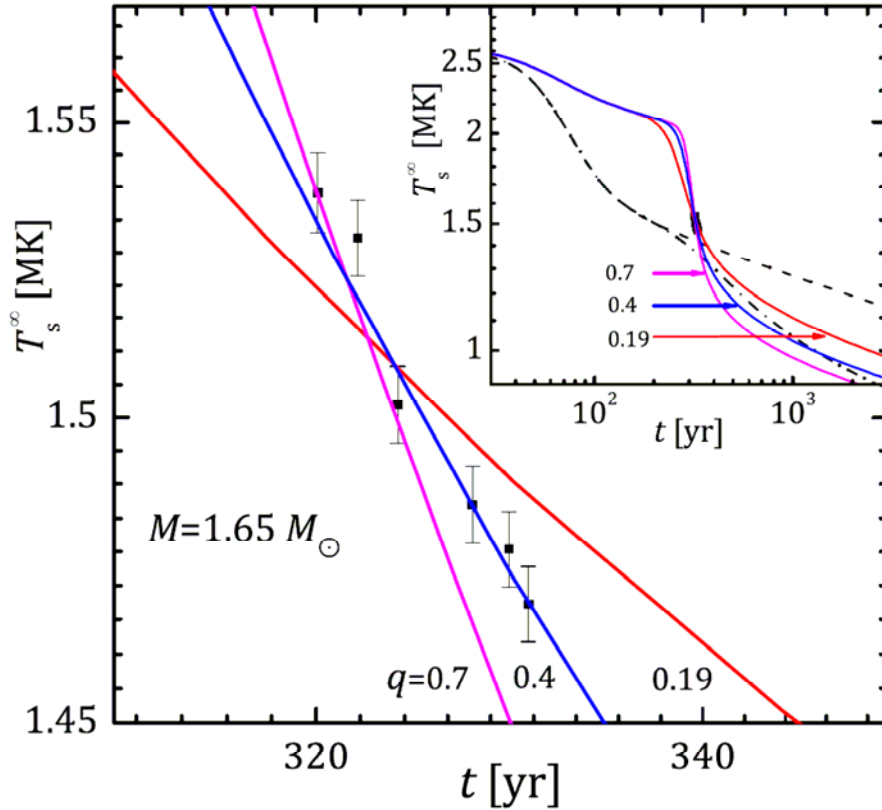


FIG. 3. A typical good fit to Cas A’s rapid cooling for a  $1.4M_{\odot}$  star, built from the EOS of APR [25] with an envelope mass  $\Delta M_{\text{light}} = 5 \times 10^{-13}M_{\odot}$ . The two dotted curves, with indicated values of  $T_C$ , are to guide the eye. The three models have a proton  $^1S_0$  gap from [26] (the model “CCDK” in [14]) which results in the entire core being superconducting. The insert shows a comparison of our results with the five data points of [7] along with their  $1\sigma$  errors.

## Superfluid Model

Rapid cooling of Cas A NS is triggered by “breaking and formation of Cooper pairs(PBF)” Neutrino-emission process.



## Superfluid Model

**Figure 3.** (Color on line) Same as on the right panel of Fig. 1 but for constant  $T_{\text{cn}}$  over the core at three values  $q = 0.19$  ( $T_{\text{cn}} = 7.55 \times 10^8$  K),  $0.4$  ( $7.2 \times 10^8$  K) and  $0.7$  ( $7 \times 10^8$  K). The inset shows the same cooling curves but over larger range of ages, together with the dashed curve for non-superfluid star and the dash-and-dot curve for the star without proton superfluidity but with neutron superfluidity at  $T_{\text{cn}} = 4.3 \times 10^8$  K.

- ◆ [\[Science News\]](#) 2011.2.4

Supernova to superfluid

- ◆ [\[New Scientist\]](#) 2011.2.4

Neutron star seen forming exotic new state of matter

- ◆ [\[NASA press release\]](#) 2011.2.23

NASA'S Chandra Finds Superfluid in Neutron star's core

- ◆ [\[Nature news\]](#) 2011.4.1

Superfluid state for Galaxy's youngest neutron star?

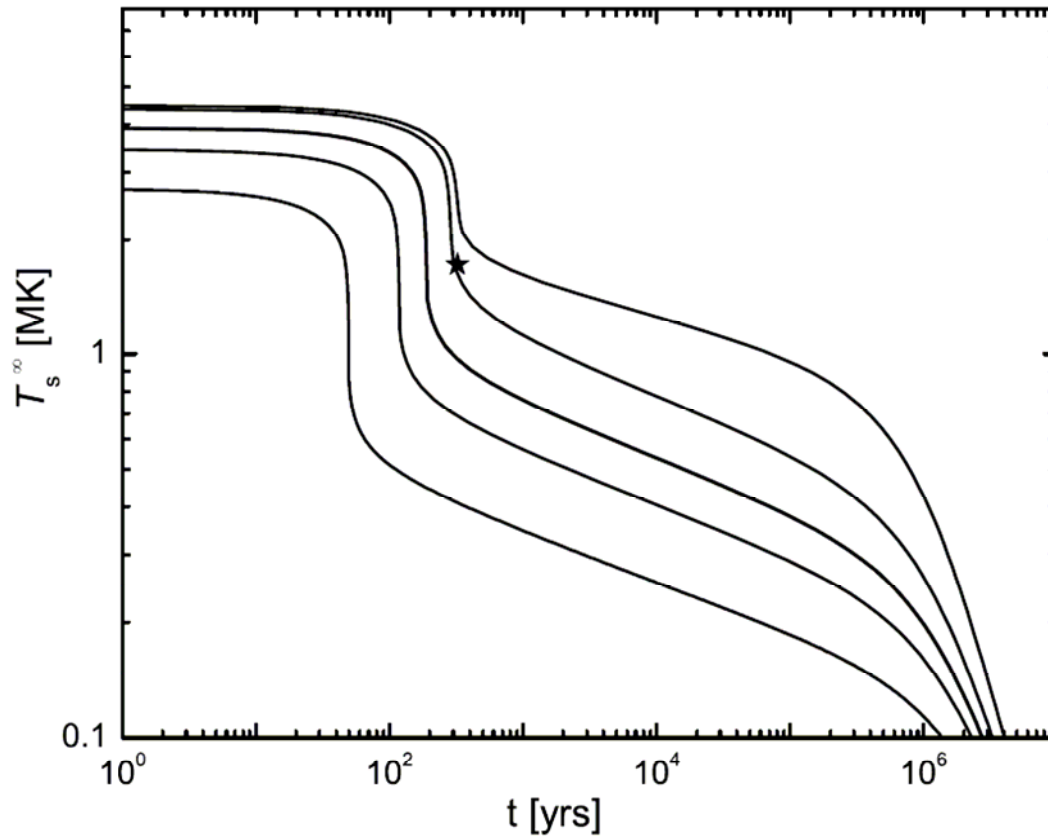


Fig. 1.— Cooling curves of neutron stars with  $K = 2$ . The curves correspond to the NS mass  $1.360M_{\odot}$ ,  $1.361M_{\odot}$ ,  $1.362M_{\odot}$ ,  $1.365M_{\odot}$  and  $1.4M_{\odot}$ , respectively. The pentagram presents the location of the observed cooling data of Cas A NS.

**The rapid cooling of Cas A NS suggested that the star is experiencing the recovery period following the r-mode heating process.**

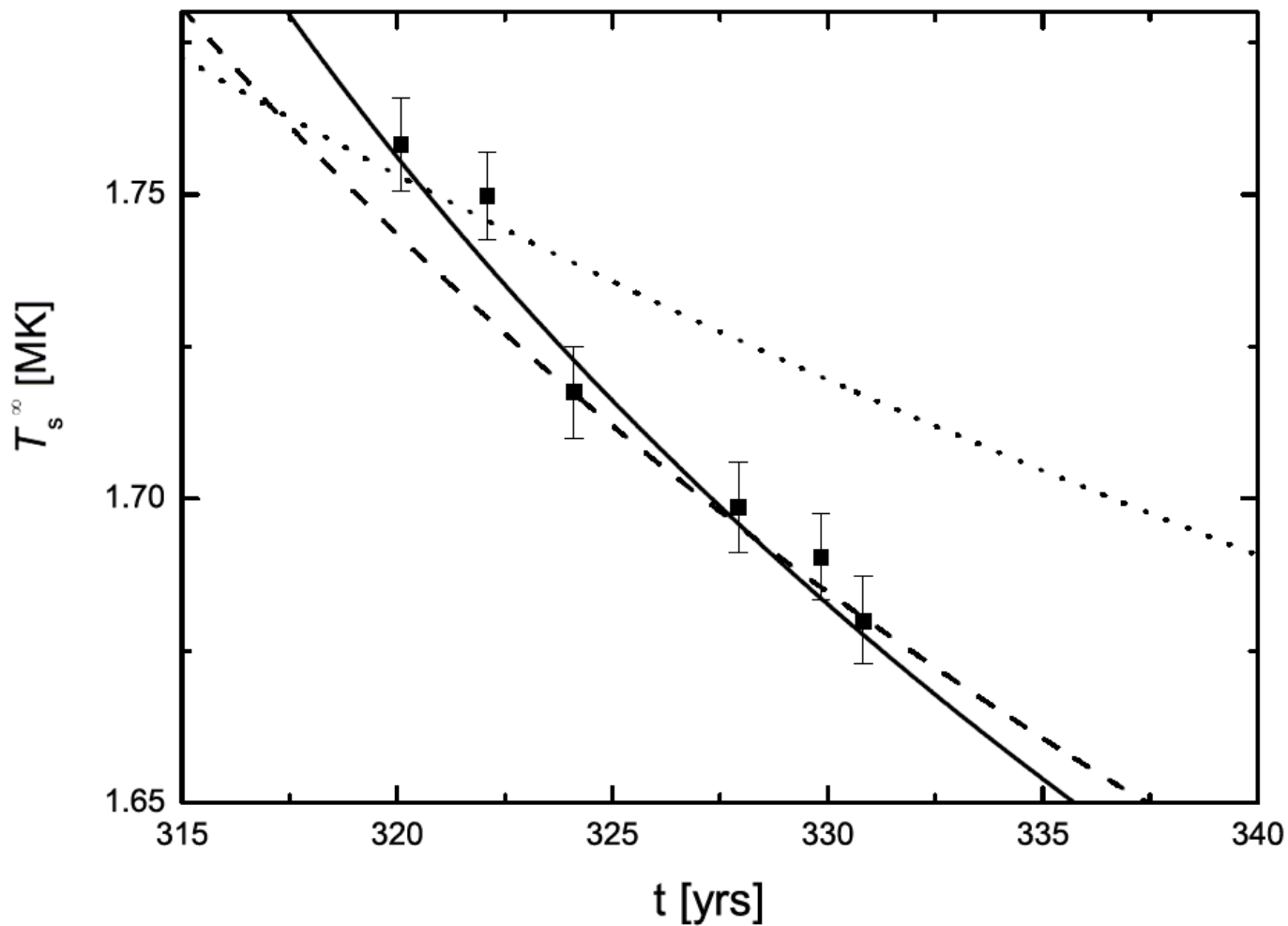


Fig. 2.— Cooling curves of the  $1.361M_\odot$  neutron star. The dot, dashed and solid curves correspond to  $K = 1.5$ ,  $K = 2.1$  and  $K = 2.3$ , respectively.

*S H Yang, C M Pi, X P Zheng, [arXiv:1103.1092](https://arxiv.org/abs/1103.1092)*

# Comparison between Two Models

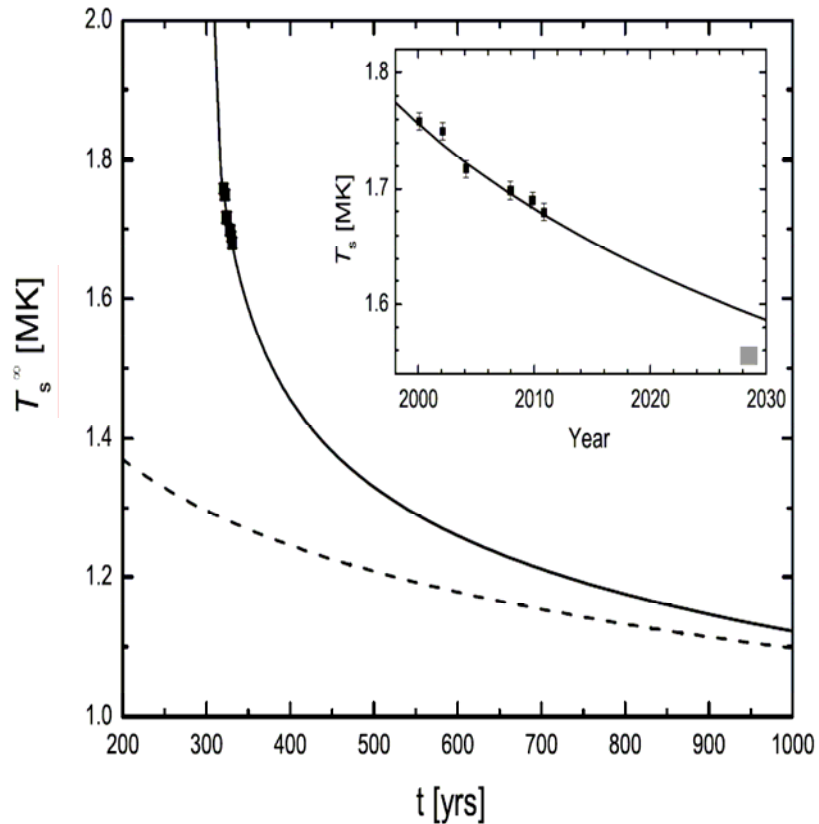


Fig. 2.— Cooling curves of the  $1.361M_{\odot}$  neutron star with  $K = 2.3$  (solid line). For comparison, the dashed line is calculated without the  $r$ -mode heating effect. The insert shows the temperature evolution in the following twenty years and the grey rectangle indicates the possible temperature scope predicted by the neutron-superfluidity-triggering model.

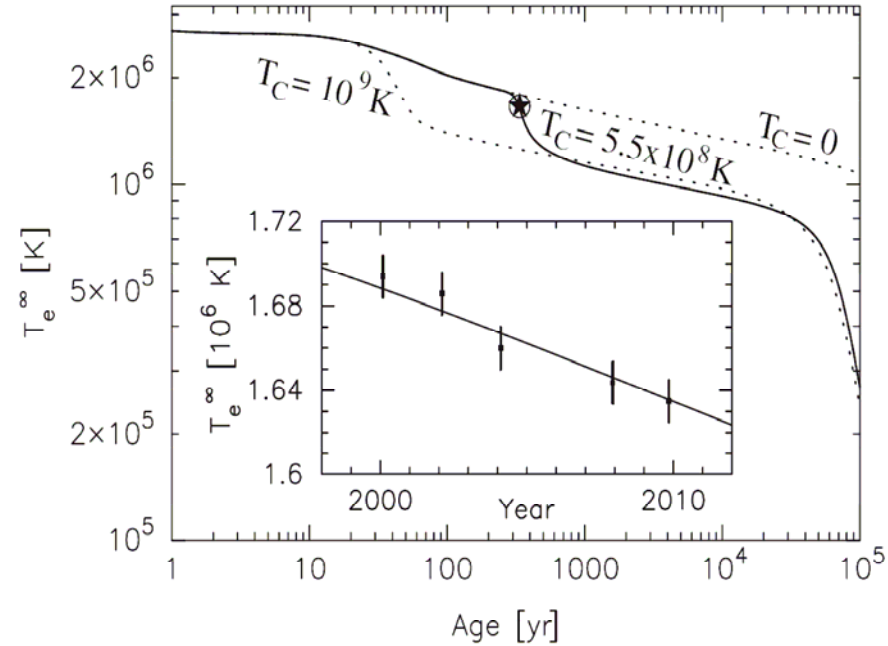


FIG. 3. A typical good fit to Cas A’s rapid cooling for a  $1.4M_{\odot}$  star, built from the EOS of APR [25] with an envelope mass  $\Delta M_{\text{light}} = 5 \times 10^{-13}M_{\odot}$ . The two dotted curves, with indicated values of  $T_C$ , are to guide the eye. The three models have a proton  $^1S_0$  gap from [26] (the model “CCDK” in [14]) which results in the entire core being superconducting. The insert shows a comparison of our results with the five data points of [7] along with their  $1\sigma$  errors.



## 5、 Summary

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- ◆ The heating due to r-mode damping enables us to explain the surface temperature data with NS model composed of only npe matter, without the including of superfluidity or exotic particles.
- ◆ The heating due to r-mode damping enables us to explain the rapid cooling of Cas A NS without the triggering by “breaking and formation of Cooper pairs(PBF)” Neutrino-emission process.
- ◆ Thermal emission is closely related to compositions and internal physics of neutron stars, which is believed to become future probe into NSs.

***Thank you!***